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An Analysis of
Opportunistic Maintenance Policy
FOR the
F100PW100 Aircraft Engine

FINAL REPORT

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APR 9 1981

Prepared by:

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screening approach. The evaluation is accomplished through a simulation of the 20-year life cycle of the F100 engine. The evaluation demonstrates the economic and performance advantages of the CPL screening policy. Model assumptions include independent part failures and exponential failure distributions for parts without life limits. Further investigation of the impact of the assumptions is suggested.

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An Analysis of Opportunistic Maintenance Policy
For the F100PW100 Aircraft Engine

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Abstract

→ An analytic model for computing optimal screening intervals for replacing life-limited parts in the F100PW100 aircraft engine is presented. The model involves determining the point in advance of a part life limit where the marginal cost of replacing a part equals the marginal expected cost of not replacing the part. The policy results in a set of Conditional Part Level (CPL) screens conditioned on the status of the module and engine at the time of engine removal. The policy is evaluated through comparison with a base/depot screening approach. The evaluation is accomplished through a simulation of the 20-year life cycle of the F100 engine. The evaluation demonstrates the economic and performance advantages of the CPL screening policy. Model assumptions include independent part failures and exponential failure distributions for parts without life limits. Further investigation of the impact of the assumptions is suggested.

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I. INTRODUCTION

Military weapon systems have become increasingly complex in recent years. The cost of operating and supporting these weapon systems has increased significantly as a percentage of total cost of ownership. The need for recognizing logistics support costs as early as possible is emphasized in several OMB and DOD documents (e.g., OMB A-109, DODD5000.1, 5000.2--System Acquisition; DODD5000.39--Integrated Logistics Support; and DODD5000.40--Reliability and Maintainability).

The United States Air Force is currently acquiring a fleet of fighter aircraft, the F-15 Eagle, which will represent a vital part of our nation's air defense system. This aircraft, manufactured by the McDonnell-Douglas Corporation, is a complex aggregation of highly integrated components, each requiring sophisticated support systems and procedures. A major component of this aircraft is its propulsion system, the Pratt and Whitney F100PW100 jet engine. This engine also powers the US Navy F-16 jet fighter.

The F100PW100 (or F100) engine consists of a set of relatively independent modules which are designed to be interchangeable among engines. Each module is made up of parts which are replaced or repaired upon failure or when maximum operating times are reached. Maximum operating times (MOT's) are assigned to critical parts which may cause catastrophic failures. A schematic of the F100 Engine is shown in Figure 1.

The support system for the F100 engine which was integrated into the design and acquisition of the F-15 fleet included a maintenance concept known as On Condition Maintenance (OCM). The OCM policy allows engine maintenance actions only when the condition of the engine

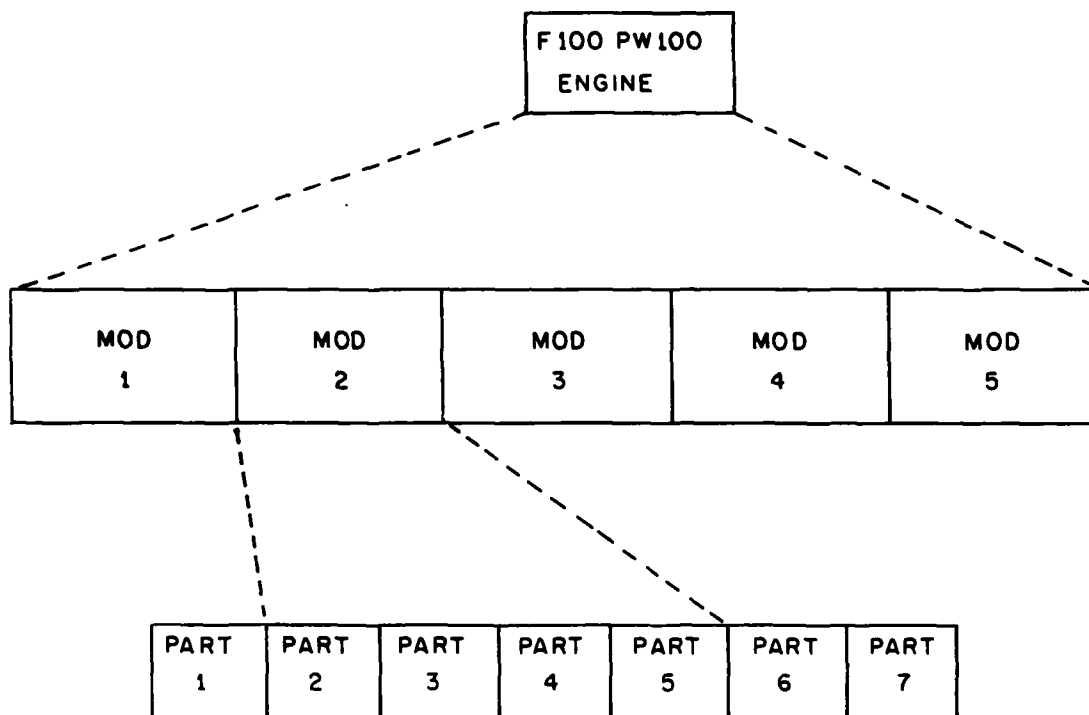


Figure 1: F100PW100 Engine/Module/Part Relationships

requires such action. This policy allows no scheduled inspections or overhauls based solely on time based criteria. Maintenance actions are initiated for one of two reasons: (1) the MOT on a life limited part is reached, or (2) a part fails. The OCM policy allows only repair or replacement of the affected component(s) with no action taken on other engine components at the time of engine removal.

The potential impact of this policy on system performance and support cost is evident. A maintenance policy which disallows "looking ahead" to impending maintenance actions will tend to increase the number of engine removals (and thus cost) driven by MOT's. This consequence suggests that some criteria other than that of the strict OCM policy may be appropriate. At this point, a review of possible causes for taking maintenance actions is helpful.

Maintenance actions may be taken for a number of reasons. Among these reasons are

1. Component failure
2. Suspected component defects
3. Potential safety hazard
4. Potential operational consequences
5. Economic considerations

The first three reasons are direct consequences of engineering design decisions. Component failure rates (observable failures) are functions of material characteristics, environmental conditions, and design considerations. Suspected component defects appear as secondary causes for removal and are affected by similar factors plus the availability and reliability of component testing equipment and procedures. Potential safety hazards are reduced by placing conservative MOT's on all parts likely to cause catastrophic failures. Again, the selected MOT's are functions of engineering design and operating environment

considerations which cause maintenance action to occur.

The fourth reason for taking maintenance actions is based primarily on mission requirements. An aircraft may be capable of flying safely from point A to point B, but if a specific mission requires the aircraft to complete a prescribed number of sorties at specified flight speed and altitude, certain maintenance actions may be required. Thus the operational (or mission specific) consequences of not taking maintenance actions may cause actions to be taken. The operational criteria for engine maintenance policy may involve maximum and/or minimum time between removal, maximum removals per thousand engine flying hours, or engine NRTS rates.

The final reason for maintenance action is based on an integration of performance requirements, design characteristics, and economic considerations in such a manner as to minimize life cycle maintenance cost. At times maintenance actions which are not required may be taken in order to avoid some future costs while maintaining performance specifications. Determining when maintenance actions of this type should be taken is a difficult task. All major cost factors, component reliability data, and part life-limits must be considered in the context of the existing support systems and tactical requirements for the weapon system under study. Thus economic considerations can be structured such that they integrate all other causes for maintenance actions and therefore drive the maintenance policy for a specific weapon system.

The OCM maintenance concept used in the F100 engine design addresses primarily the design related reasons for maintenance actions. A comprehensive maintenance policy for this engine must meet tactical requirements while minimizing life cycle maintenance cost. This

report describes an approach to developing a comprehensive maintenance model of this type.

II. BACKGROUND

In August 1976, the Directorate of Propulsion Systems, AFLC/LOP, requested a study concerning maintenance procedures for the F100PW100 engine. The motivation for this request was the desire to identify a maintenance policy which minimizes the long run maintenance cost of the F100 engine. The project was undertaken by the Directorate of Management Sciences, AFLC/XRS and, under the direction of Mr. John L. Madden, resulted in the development of a comprehensive and detailed computer simulation model of the F100 life cycle. The model was developed to investigate the effects of various maintenance policy decisions on F100 engine life cycle cost.

Due to the nature of the F100 engine, a maintenance policy involving "opportunistic" maintenance actions was investigated. The term "opportunistic" was used because the policies directed that certain maintenance actions take place at times when the engine was out of service for other reasons--i.e., use the removal as an opportunity to take additional actions. The initial models involved application of a "screening interval" to all life-limited parts in the engine. A screening interval is the period in advance of a part MOT during which a part is replaced if the engine is removed for other reasons. Thus an MOT-driven removal shortly after an engine removal is avoided.

This project was further advanced during the 1979 Summer Faculty Research Program (SFRP) sponsored by the Air Force Systems Command. The joint AFLC/XRS - SFRP effort resulted in an analytical model which could be used to identify a near optimal set of base and depot level

screening intervals for each life-limited part in the F100 engine (see reference 5). While these efforts made significant progress toward developing a comprehensive maintenance policy for the F100 engine, some additional study was required in order to test and further develop the analytical models so that an operational policy could be prescribed.

III. OBJECTIVES

The specific objectives of the study presented in this report were

- 1) To update and further refine the economic decision criteria screening model developed during the 1979 Summer Faculty Research Program at AFLC/XRS;
- 2) To determine the impact of an inspection plan on measures of interest and to determine if an inspection plan should be applied periodically or opportunistically; and
- 3) To assess the impact of a spares inventory with known age distribution and to develop a method for determining how parts or modules should be matched with modules or engines in order to optimize measures of interest.

The first objective is thoroughly treated in this report. An inspection plan is incorporated into the model and applied opportunistically to determine its impact on measures of system performance. The inspection intervals used are those recommended by AFLC/LOP. Since this study resulted in substantial modifications to the original economic decision screening model, the investigation of the spares inventory problem was necessarily deferred to a future project.

IV. MODEL FORMULATION

Developing a maintenance strategy which considers the economic consequences of maintenance actions requires recognition of the

costs and relationships involved in maintenance decisions. Figure 2 illustrates the maintenance system which supports the F100 engine and Table 1 shows the associated cost of maintenance decisions. The total cost of maintaining a single F100 engine over its entire life cycle can be expressed symbolically as:

$$Z = a_1 E_1 + a_2 E_2 + \sum_{k=1}^3 \sum_{j=1}^m b_{jk} M_{jk} + \sum_{i=1}^n c_i P_i + dB$$

where Z = total life cycle maintenance cost

E_1 = the number of times the engine is base reparable

E_2 = the number of times the engine is depot reparable

M_{j1} = the number of times module j is base reparable, $j=1, \dots, m$.

M_{j2} = the number of times module j is sent to the depot alone for repair, $j=1, \dots, m$.

M_{j3} = the number of times module j is sent to the depot with the engine for repair, $j=1, \dots, m$.

P_i = the number of times part i is replaced, $i=1, \dots, n$.

B = the number of base manhours required for removing and replacing engines and modules.

The a 's, b 's, c 's, and d are cost coefficients containing the appropriate cost factors from Table 1 involved in each event captured in the maintenance cost function. Since all major events and costs associated with maintenance decisions are included in this cost function, it can be used to monitor the costs and performance of alternative maintenance policies. The present worth of alternative policies is obtained by applying appropriate discount factors to costs as they are incurred.

The cost trade-off involved in determining the optimal set of maintenance actions to take at a given engine removal can be

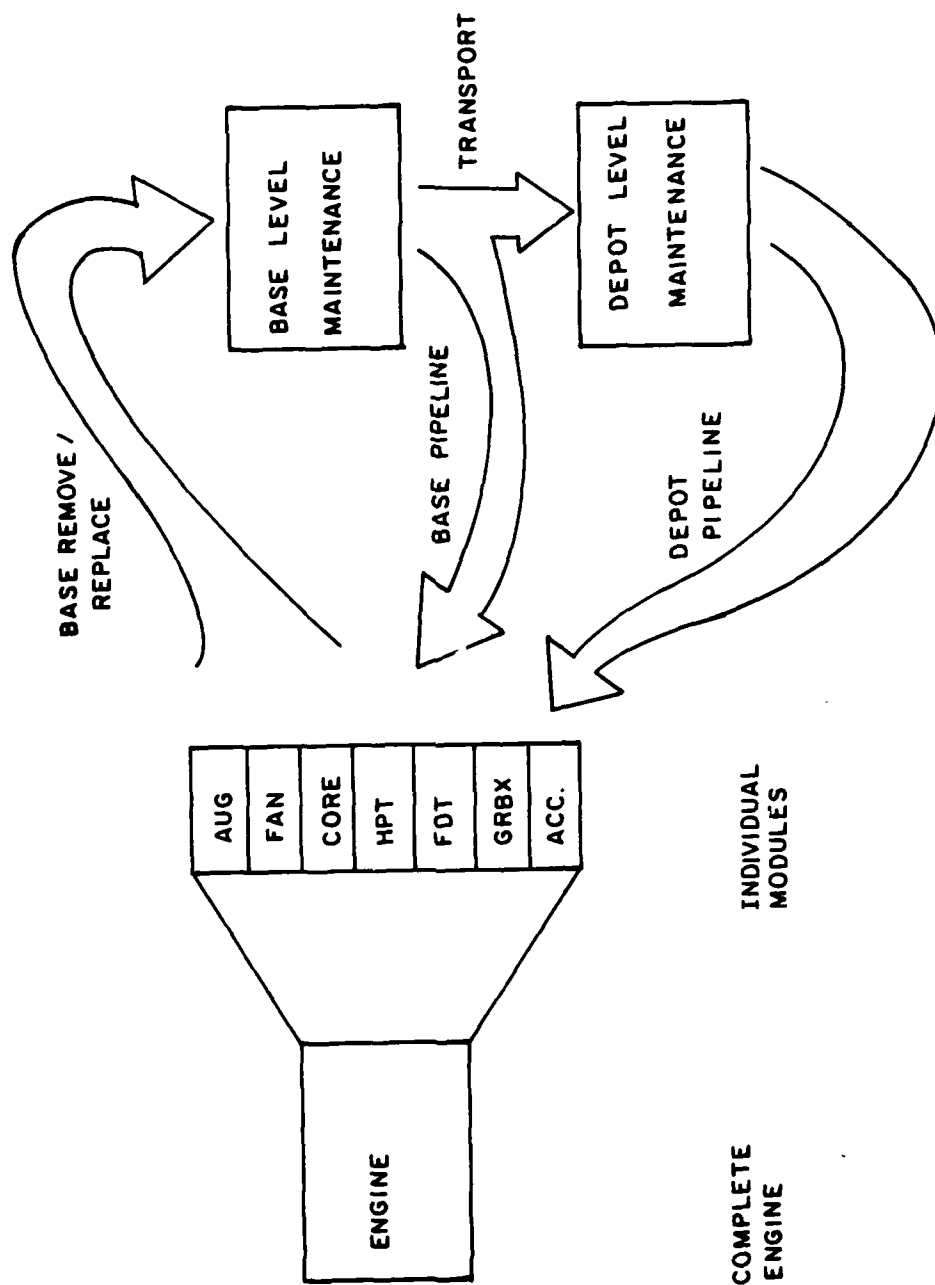


Figure 2: Schematic of the F100PW100 Maintenance System

Table 1: Cost Associated with F100 Engine Maintenance Decisions

| LEVEL | LOCATION | |
|--------|--|--|
| | BASE | DEPOT |
| Part | | replacement throwaway |
| Module | remove/replace (labor) pipeline (spares) maintenance (labor) | transportation pipeline (spares) maintenance (labor) |
| Engine | remove/replace (labor) pipeline (spares) testing (labor) | transportation pipeline (spares) maintenance (labor) |

expressed as that of identifying an optimal screening interval, t_i for each part such that

$$\boxed{\text{The cost of replacing part } i \text{ } t_i \text{ periods prior to its MOT}} = \boxed{\text{The expected cost of not replacing part } i \text{ } t_i \text{ periods prior to its MOT.}} \quad (1)$$

The left side of (1) includes the marginal cost of replacing part i early given that an engine removal has occurred. That is, only the additional cost incurred as a direct result of electing to replace part i t_i periods early is included. Since the condition of the engine and/or module determines level of repair required, the marginal cost of replacing parts early is a function of engine and module status at the time of removal. Table 2 shows the marginal cost as a function of engine and module status. The marginal cost of replacing a part early can range from a minimum of the value of the part life remaining (i.e., the throwaway cost) when the module in which the part resides is NRTS, to a maximum of the throwaway cost plus all depot related module maintenance cost when the engine is RTS and the module has no other cause for removal. In the former case all depot level maintenance cost are already incurred while in the latter case screening the part causes these costs to be incurred. For each of the cases shown in Table 2, the appropriate cost for each life-limited part and t_i value can be computed. Since the status of the engine and each module is known at the time of removal, the left side of (1) can be computed with near certainty before maintenance actions are taken.

The right side of (1) also involves the marginal cost associated with a maintenance action. This cost, however, is an expected cost and is based on the set of outcomes which may occur if part i is

Table 2: Marginal Cost of Replacing Part i t. Periods
Prior to its Life Limit as A Function of Engine
and Module Status

| LEVEL | COST FACTOR | Engine RTS° | | | Engine NRTS° | |
|--------|-------------------|-------------|---------|----------|--------------|----------|
| | | Mod OK° | Mod RTS | Mod NRTS | Mod OK | Mod NRTS |
| part | Throw away | X | X | X | X | X |
| | Remove/replace | X | | | | |
| | Base pipeline | | * | | | |
| module | Base maintenance | | * | | | |
| | Transportation | X | X | | | |
| | Depot pipeline | X | X | | | |
| | Depot maintenance | X | X | | | |

°RTS = reparable this station (base reparable)

NRTS = not reparable this station (depot reparable)

OK = no cause for removal

* Subtract these costs from marked (X) cost to obtain marginal cost of screening RTS module to depot.

not replaced t_i periods prior to its life limit. Evaluation of this cost is somewhat more complex than the left side of (1). Here, each possible outcome must be identified, the cost of each outcome must be computed, and the probability of each outcome must be determined. Relationships between alternative outcomes must be assessed to determine whether or not they are related. An expected cost must then be computed for each t_i for each part by summing the product of costs and probabilities.

Figure 3 is a tree structure which defines the potential consequences of not replacing part i during an engine removal t_i periods prior to the part's life limit. This structure is constructed and evaluated assuming that parts within a module are considered in order of increasing time until life-limit. That is, part i is considered for screening t_i periods prior to its life limit only after parts with t_i or less periods remaining have been screened. If a part with less than t_i periods is not to be replaced, the module under consideration must receive maintenance within t_i periods and thus parts with t_i or greater periods remaining are not removed. With this in mind, the alternative outcomes are analyzed as follows.

At the engine level, one of two things can occur within the next t_i periods: (1) one or more engine removals occur, or (2) no engine removals occur. If one or more engine removals occur within t_i periods, then opportunities are available for replacing part i prior to its life limit and therefore the cost of not replacing the part is reduced. If, however, no engine removals occur within the next t_i periods (case I in Figure 3), part i will cause

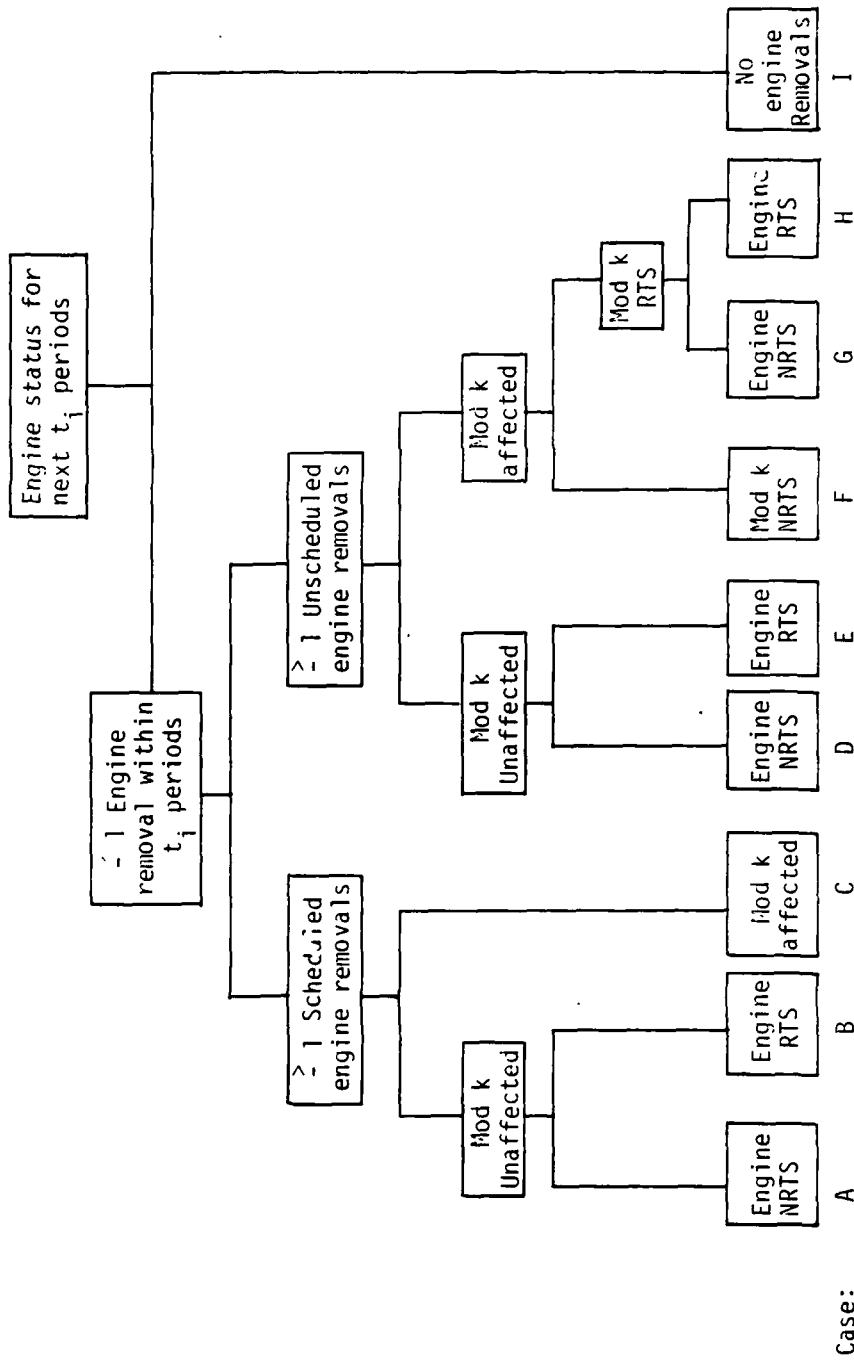


Figure 3: Possible Consequences of Not Replacing Part i t_i Periods Prior to its Life-Limit

an engine removal in exactly t_i periods and therefore cause the costs associated with the engine, module, and part level maintenance actions.

Consider the case where one or more engine removals occur within t_i periods (see Figure 3). Engine removals occur for one of two primary causes: (1) a part life-limit is reached, or (2) a part failure occurs. If a part life-limit occurs, it must occur in a module other than that of part i since parts are considered within modules in order of increasing time until life-limit. Therefore, if an MOT is the primary cause for the next engine removal, the module in which part i resides (module k) cannot be affected. Thus Case C in Figure 3 cannot occur. Case A or B occur when a scheduled engine removal occurs within t_i periods. Case B implies the engine was base repairable (although the affected module was NRTS) and thus module k does not receive depot level maintenance unless one or more parts are screened. If screening occurs, all module and part level maintenance, transportation, and pipeline cost must be attributed to the screening decision. Case A implies the engine was NRTS (depot repairable) thus all modules were transported to depot and the associated transportation and pipeline cost are already incurred. Part screening in unaffected modules would cause only depot maintenance cost to be incurred.

Another set of outcomes may occur if one or more unscheduled removals occur within the next t_i periods. Since module k is assumed to contain some parts which fail periodically, module k may be affected (i.e., failure occurs in module k) or unaffected (i.e., failure occurs in some other module). Cases D and E are analogous to Cases A and B in that the module of the part under study (module k)

is unaffected and the engine is either base or depot reparable. In Case F, module k experiences a failure which requires depot maintenance, thus the only cost associated with screening additional parts at that time is the part throwaway cost. Similarly, Case G involves a module which, although base reparable, arrives at depot as part of an engine NRTS. Case H involves a base reparable engine in which module k experiences a failure which is base reparable.

After identifying possible consequences of not replacing part i t_i periods prior to life-limit, the cost associated with replacing part i in the event that outcome occurs can be computed. These costs represent the approximate costs incurred if part i is not replaced during an engine removal t_i periods prior to its life-limit but is replaced when the outcome under consideration occurs. Table 3 shows the individual cost components which would be associated with screening decisions for each of the A through I cases shown in Figure 3. The cost codes in Table 3 are for use later in computing expected cost.

The next step in determining the expected cost of not replacing part i t_i periods prior to its life limit is to determine the probability that each of the A through I cases will occur. The Appendix provides a brief discussion of the relationships and notation used in determining these probabilities. Let

t_i = the time until life limit for the part under study in module k

$P_s(t_i)$ = probability that no part failures occur over the next t_i periods,

$P_k(t_i)$ = probability that no part failures occur in module k over the next t_i periods,

Table 3: Potential Cost Consequences of Not Replacing Part i
t_i Period Prior to its Life Limit

| <u>Code</u> | <u>Level</u> | <u>Cost Factor</u> | <u>Cases°</u> | | | | | | | | |
|----------------|--------------|--------------------|---------------|----------|----------|----------|----------|----------|----------|----------|----------|
| | | | <u>A</u> | <u>B</u> | <u>C</u> | <u>D</u> | <u>E</u> | <u>F</u> | <u>G</u> | <u>H</u> | <u>I</u> |
| C ₁ | Module | Remove/Replace | | x | | | x | | | | x |
| C ₂ | | Base Pipeline | | | | | | | | * | |
| C ₃ | | Base Maintenance | | | | | | | | * | |
| C ₄ | | Depot Transport | | x | | | x | | | x | x |
| C ₅ | | Depot Pipeline | | x | | | x | | | x | x |
| C ₆ | | Depot Maintenance | x | x | | x | x | | | x | x |
| C ₇ | Engine | Base Maintenance | | | | | | | | | x |
| C ₈ | | Base Pipeline | | | | | | | | | x |
| C ₉ | | Base Testing | | | | | | | | | x |

°See Figure 3 for descriptions of Cases A through I.

*Subtract these cost from marked (x) cost in case H to obtain additional cost due to screening.

T_k = the minimum MOT in module k

$p(j)$ = probability that outcome j (Figure 3) occurs,
j = A,B,...,I

N_s = probability of an engine NRTS when an unscheduled removal occurs

N_k = probability that module k is NRTS when a part failure occurs in module k

The probability of each outcome in Figure 3 depends upon the individual failure distribution of each part and on the relationships among failures in different modules. Some types of failure are known to affect parts in more than one module (e.g., foreign object damage, engine over temps) while others affect only one module. The existing reliability and maintainability data bases do not yet contain sufficient data to accurately determine part failure distributions nor are they organized such that meaningful relationships regarding dependencies among module failures can be developed. Consequently some assumptions are made to facilitate computations of the necessary outcome probabilities. These assumptions are:

- (1) Part level time to failure distributions are not time dependent.
- (2) Parts within and among modules fail independently.
- (3) Outcome probabilities are based on "the next" removal rather than all removals over the next t_i periods.

The effect of (3) is to assume that the cost of replacing a part during the current removal will be evaluated against the cost of replacing that part at the next removal. This allows the possible outcomes to be associated with either a scheduled removal, an unscheduled removal, or no removal. Multiple events in the next t_i periods are not considered (except, of course, as part of the analysis when "the next" removal occurs).

Consider first the case of the next removal caused by an MOT in a part in another module. As previously mentioned, a scheduled removal due to a part in module k cannot occur within the t_i periods since parts are considered in order of increasing time until life-limit. This implies that $t_i \leq T_k$ since no part with more time remaining than the part in that module with minimum MOT is ever considered for screening. Outcome C, then, from Figure 3, cannot occur; that is, $p(C)=0$. Outcomes A and B are functions of the engine configuration after all maintenance actions have been taken, thus their probabilities are not known at the time screening decisions are made.

The intent of the "opportunistic" maintenance concept is to use unscheduled engine removals as "opportunities" to perform scheduled maintenance actions. Thus as the policy improves, within the constraints of performance requirements, the number of scheduled engine removals should be minimized. In fact, analysis of the preliminary results of the method presented in this report indicates that scheduled engine removals can be reduced to less than 20% of the total engine removals. Consequently, attention was focused on the outcomes related to unscheduled engine removals. The impact of this strategy is to group Cases A and B with Case I, no engine removals. From a cost standpoint, this approach has a minimal effect since Case A occurs very infrequently ($< 1\%$ of all removals) and the cost associated with Case B differs from that of Case I by only the base level engine maintenance, pipeline, and testing costs. By ignoring Cases A and B the expected cost of not replacing a part is somewhat overstated and therefore, in some cases, a part may be replaced when, economically, it should not have been replaced. This is a conservative action from the standpoint of system performance

and therefore will not degrade performance.

Cases D and E involve an unscheduled engine removal in which module k is unaffected. Since the probability that no engine removals occur over the next t_i periods (i.e., $p(I)$) is $P_s(t_i)$, the probability of at least one unscheduled removal in this interval is $1 - P_s(t_i)$ (recall $p(A) = p(B) = 0$ is assumed). If exponential failure probabilities are assumed (i.e., time independent), the probability that module k is unaffected in an unscheduled engine removal is determined as follows:

Let $P_s^k(t_i)$ = probability that all modules excluding module k survive the next t_i periods.

Then $1 - P_s^k(t_i)$ = probability that one or more modules excluding module k fail in the next t_i periods.

The probability of an unscheduled engine removal in the next t_i periods in which module k is not affected, then, is the probability module k survives and one or more other modules fail. Assuming independent failures, this probability is

$$\begin{aligned} & P_k(t_i) [1 - P_s^k(t_i)] \\ &= P_k(t_i) - P_k(t_i) P_s^k(t_i). \end{aligned}$$

But $P_k(t_i) P_s^k(t_i) = P_s(t_i)$ for exponential failures and the probability of interest can be written as

$$[P_k(t_i) - P_s(t_i)]$$

The probabilities for Cases D and E can now be computed by applying the appropriate NRTS probabilities to the above expression. For Case D,

$$p(D) = [P_k(t_i) - P_s(t_i)] N_s$$

For Case E

$$p(E) = [P_k(t_i) - P_s(t_i)] (1 - N_s)$$

Cases F, G, and H involve unscheduled removals in which module k is affected. The probability that module k is affected is simply the probability that it does not survive the next t_i periods, i.e., $1-P_k(t_i)$. This probability can be verified by showing that the probabilities for branches of Figure 3 always sum to the probability at the next highest level. Thus,

$$p(D,E) + p(F,G,H) = p(D,E,F,G,H)$$

$$\text{or } [P_k(t_i) - P_s(t_i)] + [1-P_k(t_i)] = 1-P_s(t_i).$$

Assuming engine and module useage NRTS probabilities are independent, probabilities for Cases F, G, and H are as follows. For Case F,

$$p(F) = [1-P_k(t_i)]N_k.$$

For Case G,

$$p(G) = [1-P_k(t_i)](1-N_k)N_s.$$

For Case H,

$$p(H) = [1-P_k(t_i)](1-N_k)(1-N_s).$$

At this point the outcomes, their costs, and their probabilities can be combined into an expected cost function for use in determining the optimal screening intervals for each part. Table 4 summarizes this information for each case considered. Note that this expected cost function is a linear function of the engine and module k survival probabilities. As t_i increases, both $P_s(t_i)$ and $P_k(t_i)$ decrease indicating that the expected cost of not replacing a part decreases as its time until life-limit increases. The constant term, $(1-N_s)(1-N_k) C''$, indicates that there is always some cost associated with choosing not to replace a part when the opportunity exist to do so.

Table 4: Cost and Probabilities for Potential Outcomes Due To Not Replacing Part i t_i Periods Prior to its Life Limit

| <u>Case°</u> | <u>Cost Codes*</u> | <u>Probability</u> |
|--------------|-------------------------------|------------------------------|
| A | C_6 | approximately 0 |
| B | $C_1+C_4+C_5+C_6$ | assumed 0 |
| C | --- | 0 |
| D | C_6 | $[P_k(t_i)-P_s(t_i)]N_s$ |
| E | $C_1+C_4+C_5+C_6$ | $[P_k(t_i)-P_s(t_i)](1-N_s)$ |
| F | 0 | $[1-P_k(t_i)]N_k$ |
| G | 0 | $[1-P_k(t_i)]N_s$ |
| H | $(C_4+C_5+C_6)-(C_2+C_3)$ | $[1-P_k(t_i)](1-N_k)(1-N_s)$ |
| I | $C_1+C_4+C_5+C_6+C_7+C_8+C_9$ | $[P_s(t_i)]$ |

°Cases as shown in Figure 3

*Cost codes as shown in Table 3

Since all other terms go to zero, the sums of the products of cost and probability for cases D,E,H, and I can be used to determine the expected cost of not replacing part i t_i periods prior to life limit. Simplifying, let

$$C' = (1-N_s)(C_1+C_4+C_5+C_6) + N_s C_6$$

$$C'' = (C_4+C_5+C_6) - (C_2+C_3)$$

$$C^* = C_1+C_4+C_5+C_6+C_7+C_8+C_9$$

Expected cost of not replacing part i t_i periods prior to its life limit is

$$P_s(t_i)[C^*-C'] + P_k(t_i)[C'-(1-N_s)(1-N_k)C''] + (1-N_s)(1-N_k)C''.$$

To determine the optimal screening interval, equation (1) is solved for t_i which equates the cost of replacing part i t_i periods early and the expected cost of not replacing part i t_i periods early. As previously shown, the cost of replacing a part early depends on the status of the engine and module at the time of removal and on the life remaining on the part in question. The expected cost of not replacing part i depends only on the life remaining (i.e., t_i). Thus, optimal screening intervals for each part must be computed for each of the following conditions:

| | | module k status | |
|---------------|------|-----------------|------------|
| | | Affected | Unaffected |
| Engine Status | RTS | t_{i1}^* | t_{i2}^* |
| | NRTS | t_{i3}^* | t_{i4}^* |

where t_{ij}^* = optimal screening interval for part i and condition j

The t_{ij}^* 's obtained by solving (1) are applied in a sequential manner beginning with the part in the module under consideration which has the least time remaining. Once part i in that module is reached which has more life remaining than the appropriate t_{ij}^* , no further parts are considered for screening. All t_{ij}^* 's are bounded to the range $0 \leq t_{ij}^* \leq T_k$. If the computed $t_{ij}^* > T_k$, t_{ij}^* is set equal to T_k .

V. APPLICATION:

The optimal screens developed through the model described above are referred to as Conditional Part Level (CPL) screens since the optimal screens are applied at the part level and are conditioned on engine and module status. To illustrate how CPL screens are

computed and applied, the following example is offered.

Consider the F100 fan module which consist of eight life limited parts and several parts without life limits. The input unscheduled removals per 1000 engine flying hours for the fan is 0.305 indicating a mean time between unscheduled removals for this module of 3278 engine flying hours. For the entire engine, the mean time between unscheduled removals is approximately 174 engine flying hours. Assuming exponential failures, these parameters are used to compute $P_s(t_i)$ and $P_k(t_i)$ for different values of t_i .

The input cost data available through AFLC/XRS are used to compute both the cost of replacing and not replacing part i t_i periods prior to its life limit. The cost of replacing part i t_i periods early as a function of module status and t_i includes

a) Marginal module maintenance cost:

| <u>Fan Module Status</u> | <u>Cost to Screen</u> |
|-------------------------------|-----------------------|
| 1) Depot repairable (NRTS) | \$0 |
| 2) Unaffected but Engine NRTS | 2667 |
| 3) Unaffected, Engine RTS | 5363 |
| 4) Base repairable (RTS) | 3492 |

b) Part throwaway cost (value of remaining part life):

Let SLP = part stock list price.

Then, Throwaway cost = $\frac{SLP}{MOT} (t_i)$

For the Fan Module $\frac{SLP}{MOT}$ for each part is

| <u>Part Nomenclature</u> | <u>SLP/MOT</u> |
|--------------------------|----------------|
| 303 1STG DISK | \$4.731 |
| 304 2STG DISK | 4.03 |
| 305 3STH DISK | 3.680 |
| 306 1STG SEAL | 0.407 |
| 307 FRNT SEAL | 0.243 |
| 308 REAR SEAL | 0.296 |
| 309 RETAINER | 0.164 |
| 310 2STG SEAL | 0.450 |

The cost of replacing a part early can be computed for any t_i by summing the appropriate marginal cost factor and the part throwaway cost.

The expected cost of not replacing part i is a function of $P_s(t_i)$ and $P_k(t_i)$ where k is the fan module. For exponential failures,

$$P_s(t_i) = \exp(-t_i/174)$$

$$P_k(t_i) = \exp(-t_i/3278)$$

The required cost factors are included in the expected cost equation yielding,

$$\text{Expected Cost} = (1425) \exp(-t_i/174) + (3745) \exp(-t_i/3278) + 1429$$

To find optimal screens for a single part i , say, the First Stage Disk, determine the t_i value where the cost of replacing is equal to the expected cost of not replacing. For the four module conditions previously mentioned the screens (in engine flying hours) are

determined as follows (t_{ij}^* found through binary search):

| Module Status | t_i | Cost to Replace | E(Cost Not to Replace) | t_{ij}^* |
|-------------------------------------|-------|-----------------|------------------------|------------|
| Module NRTS (j=3) | 900 | 4258 | 4283 | 904 |
| | 950 | 4498 | 4238 | |
| Unaffected but Eng NRTS (j=4) | 450 | 4796 | 4801 | 450 |
| | 500 | 5033 | 4725 | |
| Unaffected, Eng RTS (j=2) | 100 | 5836 | 5864 | 102 |
| | 150 | 6073 | 5603 | |
| Module RTS (j=1) | 300 | 4911 | 5101 | 326 |
| | 350 | 5148 | 4986 | |

In a similar manner, optimal CPL screens can be computed for each module condition for each part. For the entire fan module, the optimal screens (in engine flying hours) determined through this process are,

| Part Nomenclature | NRTS | OK/Eng NRTS | OK/Eng RTS | RTS |
|-------------------|------|-------------|------------|------|
| 303 1STG DISK | 904 | 450 | 102 | 326 |
| 304 2STG DISK | 1032 | 507 | 110 | 363 |
| 305 3STG DISK | 1113 | 542 | 114 | 386 |
| 306 1STG DISK | 1363 | 1363 | 197 | 1239 |
| 307 FRNT SEAL | 1363 | 1363 | 197 | 1363 |
| 308 REAR SEAL | 1363 | 1363 | 195 | 1363 |
| 309 RETAINGER | 1363 | 1363 | 202 | 1363 |
| 310 2STG SEAL | 1363 | 1363 | 188 | 1196 |

The minimum MOT (expressed in engine flying hours) in the fan module is 1363 EFH, thus the maximum screening interval is 1363 EFH. Note that the screens associated with a NRTS module are quite large indicating that most parts should be replaced whenever the module receives

depot maintenance. Conversely, base level screens for an unaffected module are quite small indicating that a part must be relatively near its MOT before a module can be sent to depot for screening reasons only. The other cases lie between these extremes.

The implementation of the CPL screens is relatively simple. At the base level, two screens are available for each part; at the depot level, two other screens are available for each part. When an RTS module is removed at base level, the appropriate screen is applied to the part with minimum time remaining. If the time remaining exceeds the appropriate base level screen, no further action is taken; if the time remaining is less than or equal to the appropriate screen, the module is sent to depot for maintenance and further screening. Once a module arrives at depot, the appropriate depot level screens are applied in order of increasing time to life limit and parts are replaced until the time remaining on a part exceeds its optimal CPL screen.

VI. ANALYSIS AND RESULTS:

Optimal CPL screens were computed for each life limited part in the F100 engine. The performance of the CPL screening concept was tested through a computer simulation model of the F100 20-year life cycle. The model tested the policy under conditions of no inspection are prescribed inspection intervals. Results were compared with the competing alternative policy with the best known performance at the time the CPL policy was developed. This competing policy involved single base and depot level screens applied at the part level. The base screens were set at 5% of the part MOT's. Depot screens were set at 1300 periods for unscheduled removals and 1800

periods for scheduled maintenance except for in the fan drive turbine where the screens were 1500 and 2300 periods respectively. The "periods" were expressed in the natural operating units of the part in question (e.g., total operating time, engine flying hours, or cycles).

The computer simulation model used to compare these alternatives operated under the following set of assumptions:

- (1) Modules were aged at the start of the engine life cycle by assigning to all parts within the same module an age equal to a randomly selected fraction of the minimum time to life limit or time to failure.
- (2) No part was removed which had more time remaining on it than the time remaining until the end of the engine life cycle.
- (3) For the inspection interval model, the time to failure distribution for parts without life limits was not affected by module inspections.
- (4) Modules were aged again at the beginning of each new life cycle throughout the simulation run.
- (5) The present worths of future costs were determined by discounting them from the time at which the cost was incurred back to the beginning of the life cycle; a 10% discount factor was used.

The results of this analysis are shown in Tables 5 through 8 and Figures 4 through 9. Table 5 and Figures 4 and 6 show the performance of the CPL policy without inspection intervals. Table 6 and Figures 5 and 6 show the same information for the Base/Depot screening policy with no inspections. In comparison, the CPL policy shows a significant reduction in discounted life cycle maintenance cost per engine, a reduction in the engine removals per 1000 engine flying hours, a slight reduction in base maintenance manhours per base removal, and a substantial reduction in the number of times an engine is removed within 100 engine flying hours of the last

Table 5: Model Performance Evaluation
Based on 100 Life Cycles

Model: CPL Screens without Inspections

| <u>MODULE</u> | <u>REM/KEFH</u> | <u>% NRTS</u> |
|-----------------------|-----------------|---------------|
| Engine | 6.725 | 5.65 |
| Augmentor | 1.605 | 14.35 |
| Accessories w/MOT's | 1.944 | 100.00 |
| Fan | 0.899 | 88.83 |
| Core | 0.765 | 78.85 |
| High Pressure Turbine | 1.806 | 80.05 |
| Fan Drive Turbine | 0.980 | 80.75 |
| Gearbox | 0.235 | 52.08 |
| Accessories W/O MOT's | 1.706 | 7.33 |

Base manhours/Base removal: 113.5

Cost per engine life cycle (10% discount rate): \$211,351

Table 6: Model Performance Evaluation
Based on 100 Life Cycles

Model: Base/Depot Screens without Inspections

| <u>MODULE</u> | <u>REM/KEFH</u> | <u>% NRTS</u> |
|-----------------------|-----------------|---------------|
| Engine | 7.836 | 5.57 |
| Augmentor | 1.547 | 16.16 |
| Accessories w/MOT's | 1.507 | 100.00 |
| Fan | 1.157 | 89.62 |
| Core | 1.338 | 87.00 |
| High Pressure Turbine | 2.130 | 73.89 |
| Fan Drive Turbine | 1.203 | 78.82 |
| Gearbox | 0.431 | 83.33 |
| Accessories W/O MOT's | 1.767 | 7.35 |

Base manhours/Base removal: 120.2

Cost per engine life cycle (10% discount rate): \$279,327

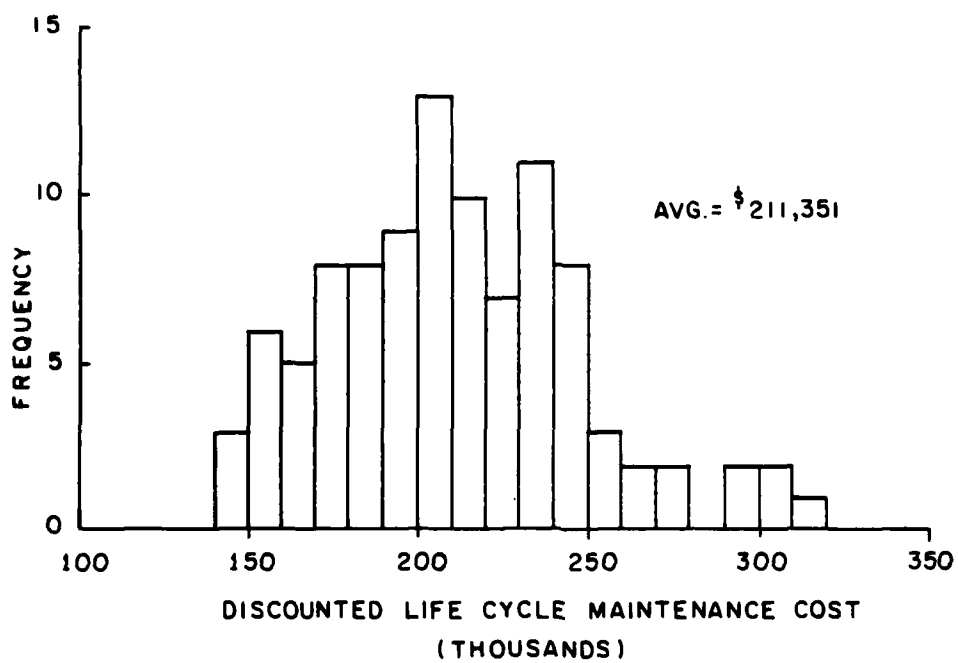


Figure 4: CPL Screens - No Inspections
(100 Life Cycles - 10% Discount Rate)

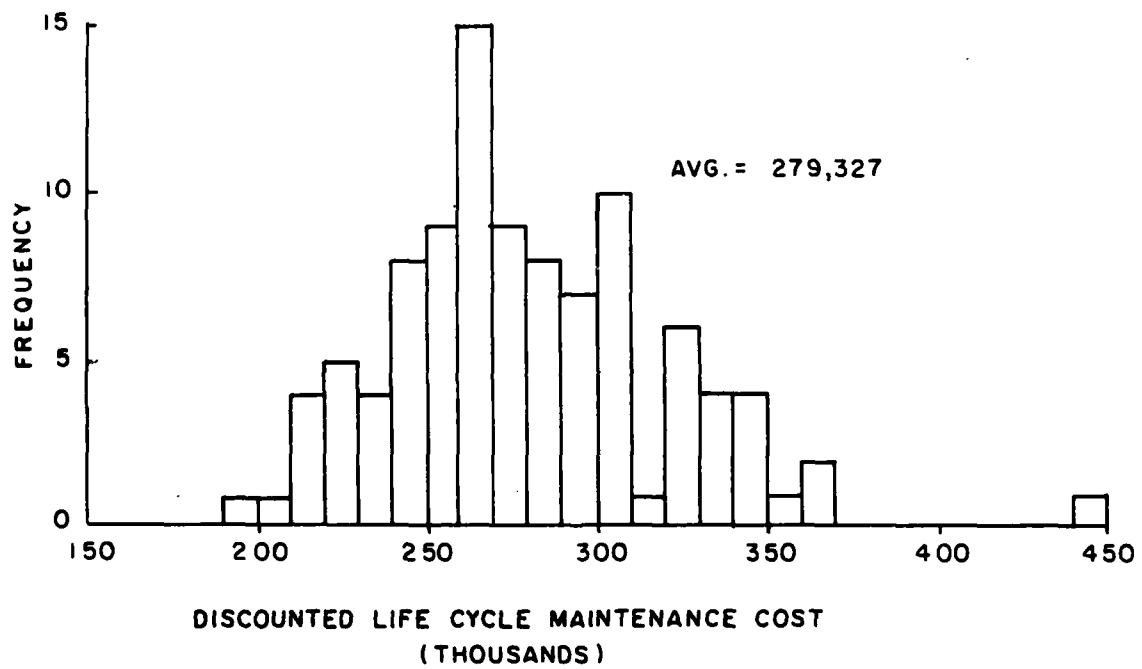


Figure 5: Base/Depot Screens - No Inspections
(100 Life Cycles - 10% Discount Rate)

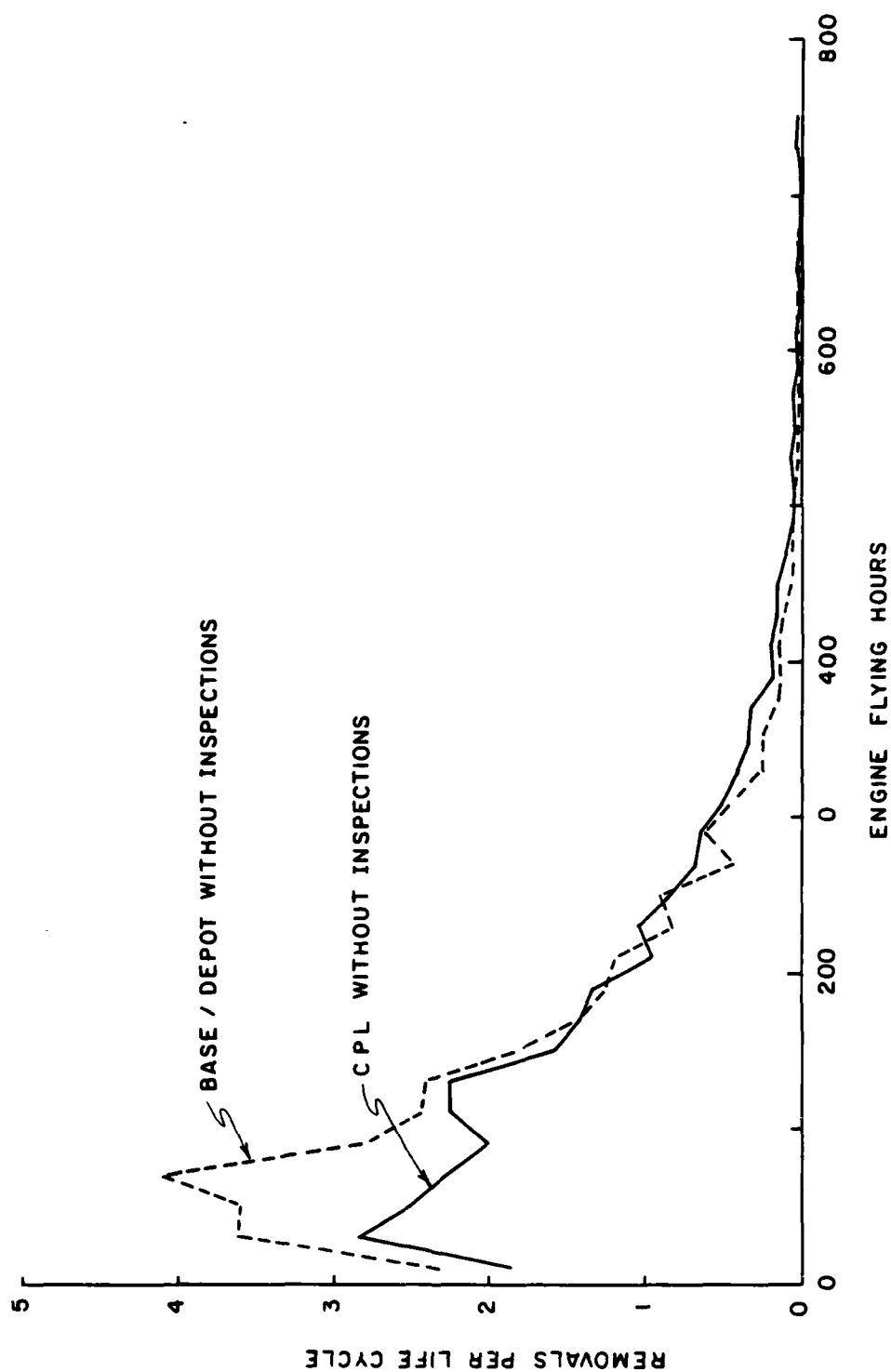


Figure 6: Distribution of Time Between Engine Removals

removal. Each of these measures is important from an economical and/or tactical viewpoint. The CPL policy, therefore, offers considerable advantage over the Base/Depot screening policy when no inspection intervals are employed.

The impact of inspection intervals is evaluated by placing an MOT on a dummy part within each module so that each module is forced to the depot for maintenance each time the inspection interval passes. The model allows inspections to be performed opportunistically so that they may be performed before an inspection interval elapses. Optimal CPL screens were re-computed for each module since the inspection intervals affected the upper bounds for the t_{ij}^* 's. The inspection intervals were set as prescribed by AFLC/LOP and XRS for use in evaluating the CPL model performance. Table 7 and Figures 7 and 9 show results with the CPL policy; Table 8 and Figures 8 and 9 show results for the Base/Depot screen policy. While the inspection policy reduced the difference in costs and reversed the base man-hour performance, a substantial difference in engine removals per 1000 engine flying hours was observed and the number of engine removals within 100 engine flying hours of the last removal heavily favored the CPL policy. In general, the CPL model performed at least as well as the Base/Depot screening model on economic measures and was generally superior on system performance measures.

VII. CONCLUSIONS AND FUTURE RESEARCH NEEDS:

The analysis presented in this report demonstrates the advantages of a policy which makes optimal use of information concerning engine and module status, part failure distributions, relevant cost parameters, and part level time to life limit data. The analysis resulted in a model which is easily resolved when changes

Table 7: Model Performance Evaluation
Based on 100 Life Cycles

Model: CPL Screens with Inspections

| <u>MODULES</u> | <u>REM/KEFH</u> | <u>% NRTS</u> |
|-----------------------|-----------------|---------------|
| Engine | 6.937 | 6.19 |
| Augmentor | 1.525 | 15.76 |
| Accessories w/MOT's | 1.926 | 100.00 |
| Fan | 1.483 | 98.68 |
| Core | 1.618 | 98.03 |
| High Pressure Turbine | 2.281 | 95.81 |
| Fan Drive Turbine | 1.225 | 94.00 |
| Gearbox | 0.620 | 86.56 |
| Accessories W/O MOT's | 1.684 | 9.02 |

Base manhours/Base removals: 136.6

Cost per engine life cycle (10% discount rate): \$268,297

Table 8: Model Performance Evaluation
Based on 100 Life Cycles

Model: Base/Depot Screens with Inspections

| <u>MODULE</u> | <u>REM/KEFH</u> | <u>% NRTS</u> |
|-----------------------|-----------------|---------------|
| Engine | 8.544 | 4.62 |
| Augmentor | 1.669 | 15.86 |
| Accessories w/MOT's | 1.473 | 100.00 |
| Fan | 1.456 | 90.74 |
| Core | 1.520 | 91.45 |
| High Pressure Turbine | 2.272 | 74.87 |
| Fan Drive Turbine | 1.243 | 79.29 |
| Gearbox | 0.539 | 79.09 |
| Accessories W/O MOT's | 1.826 | 7.11 |

Base manhours/Base removal: 119.2

Cost per engine life cycle (10% discount rate): \$279,396

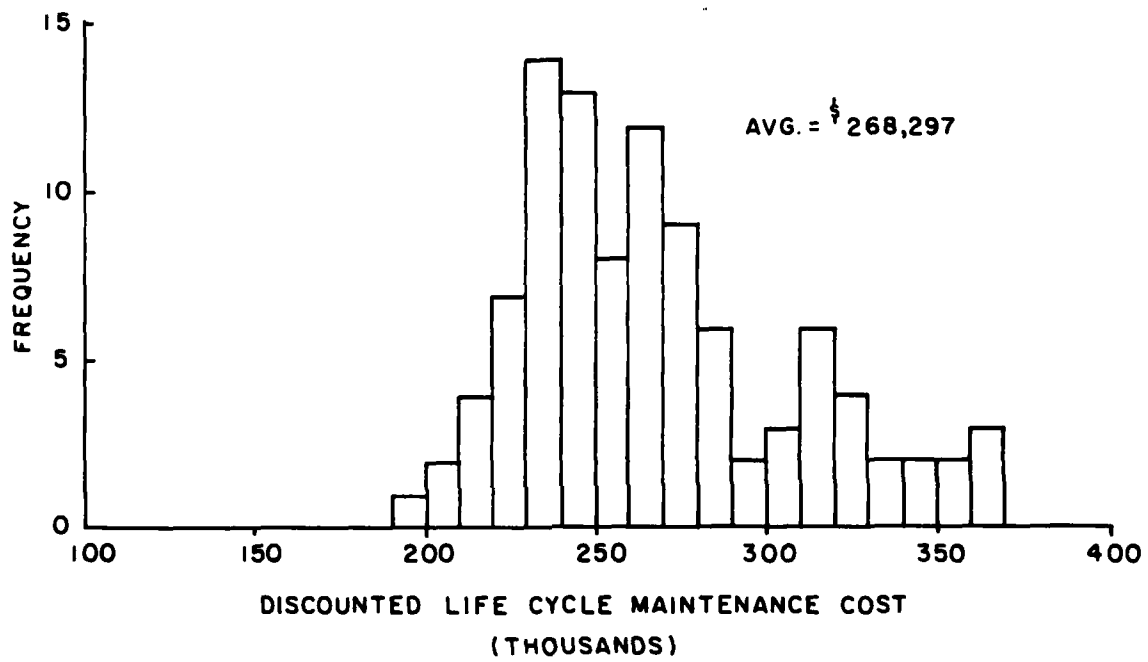


Figure 7: CPL Screens - Inspections
(100 Life Cycles - 10% Discount Rate)

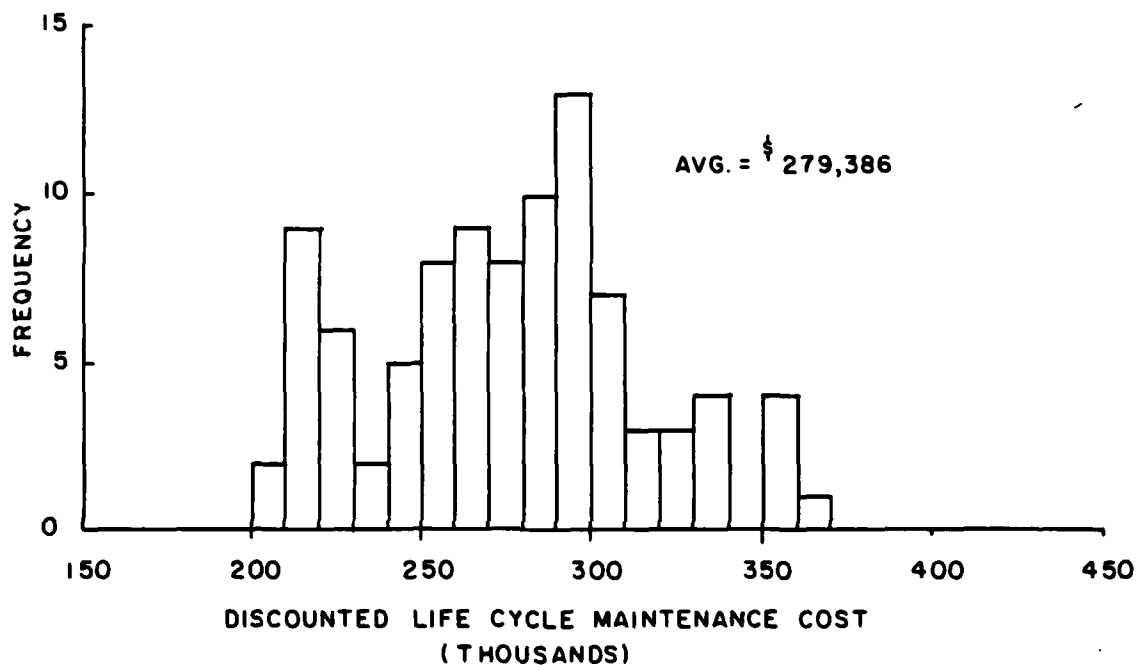


Figure 8: Base/Depot Screens - Inspection
(100 Life Cycles - 10% Discount Rate)

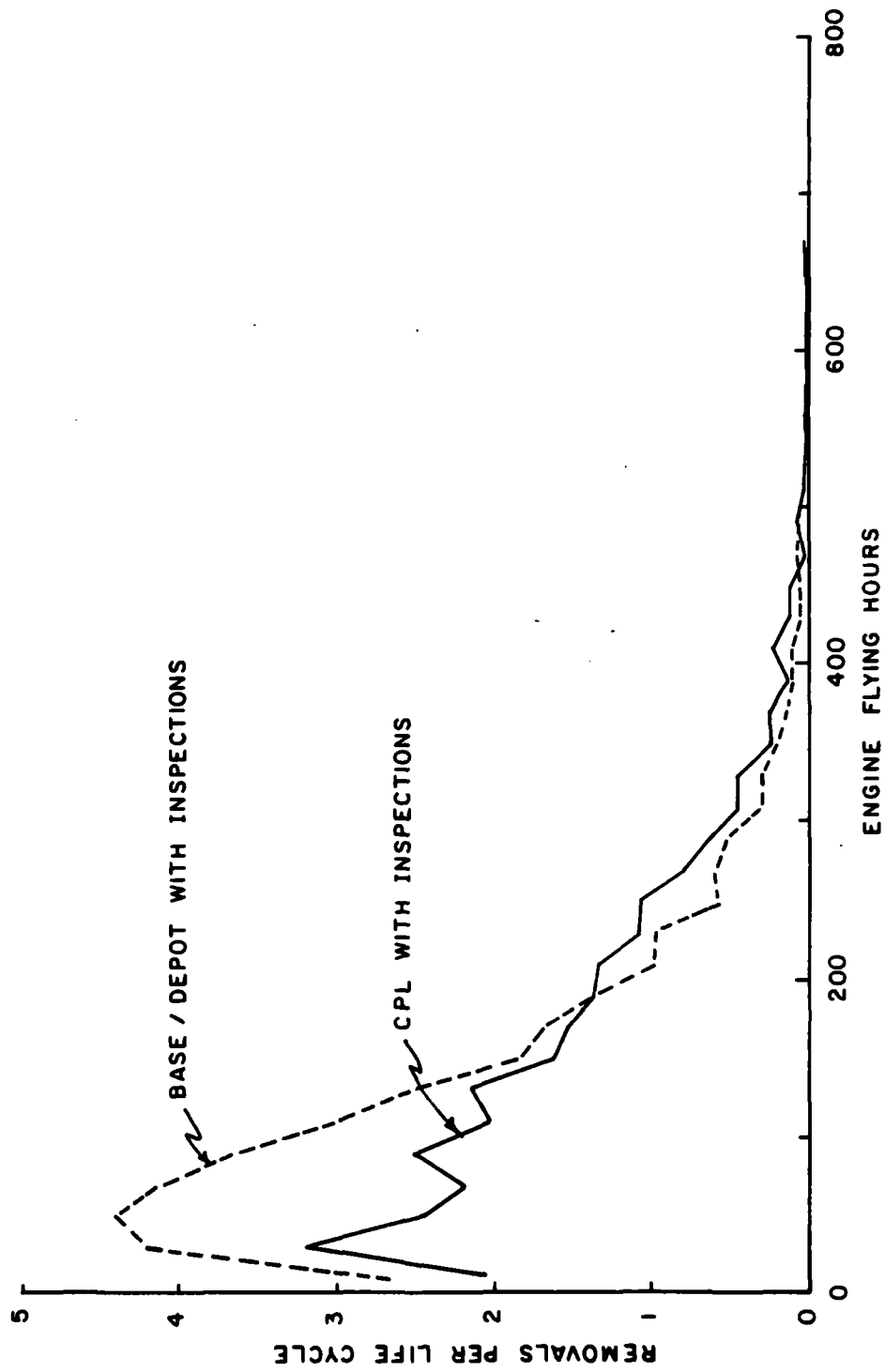


Figure 9: Distribution of Time Between Engine Removals

in input data occur. Although a computerized information system might facilitate the implementation and operation of the policy prescribed by the model, it can be implemented without such support. Since the model is analytical in nature, the computing time necessary to determine optimal screens is minimized. The solutions are optimal with respect to the input parameters and relationships rather than heuristic estimates based on simulation results.

The major weaknesses in the analysis presented herein center on the assumptions regarding part failure distributions and independence among modules. The next vital step toward future improvements in opportunistic maintenance policy for the F100 engine must come from investigating the extent to which dependencies in module removal rates affect the optimal screening policy. If unscheduled module removals are highly dependent, the number of unscheduled removals is overstated when independence is assumed and, therefore, the number of opportunities for maintenance actions is overstated. The effect of dependent module removals on the CPL screens would be to increase the screens at each level and the cost associated with screening parts. If the effect of dependent module removals is significant, appropriate data sources must be identified so that the dependencies can be quantified through conditioned probability distributions. This need may, in turn, identify the need for new data collection requirements, new data systems, and new methods for aggregating existing data.

The second type of assumption concerns the part failure distributions. Since parts were assumed to fail exponentially, the analytical formulation did not consider the age of parts in determining survival probabilities. If part failures are time dependent,

then the optimal screening intervals depend on the specific age configuration of all parts without life limits. Additionally, as parts with time dependent failure rates age, it may be advantageous to include them for consideration for opportunistic maintenance. Consequently, the analytical model based on exponential failures will require some revision in order to properly accommodate time dependent failure distributions.

While the approach presented here is not the last word in opportunistic maintenance for the F100 engine, it is another step toward an economically and tactically attractive maintenance policy. The CPL screening policy has been shown to be superior to other policies currently under investigation and therefore should receive consideration. Its greatest promise, perhaps, is the potential generalization of the approach to other propulsion systems, other aircraft components, and to entire weapon systems. With additional research, new data sources and systems, and further experience this can become a reality.

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Appendix

Reliability Concepts

A system consists of n components in series each having independent but not identical failure density functions. If any component fails, the system fails.

$R_i(t)$ is the reliability function of component i

where $R_i(t) = 1 - F_i(t) = P(T > t)$ for component i

The conditional probability that component i fails in the interval $[t, t+t_i]$ given that it has survived to time t is given by

$$\frac{R_i(t) - R_i(t+t_i)}{R_i(t)}$$

At system time t , the probability that a single component does not fail in the interval $[t, t+t_i]$ given that it survived to time t and was last replaced at t_i^0 , is

$$1 - \frac{R_i(t-t_i^0) - R_i(t-t_i^0+t_i)}{R_i(t-t_i^0)}$$

Let $t_i' = t - t_i^0$ for $i=1, \dots, n$,

$P_S(t_i)$ = probability that the system survives at least t_i periods; then,

$$\begin{aligned} P_S(t_i) &= \prod_{i=1}^n \left[1 - \frac{R_i(t_i') - R_i(t_i'+t_i)}{R_i(t_i')} \right] \\ &= \prod_{i=1}^n \left[\frac{R_i(t_i'+t_i)}{R_i(t_i')} \right] \end{aligned}$$

For Weibull failure densities with location, shape, and scale parameters of δ , β , θ respectively,

$$P_s(t_i) = \prod_{i=1}^n \frac{\exp \left[- \left(\frac{t_i' + t_i - \delta_i}{\theta_i - \delta_i} \right)^{\beta_i} \right]}{\exp \left[- \left(\frac{t_i' - \delta_i}{\theta_i - \delta_i} \right)^{\beta_i} \right]}$$

$$= \exp \left\{ \sum_{i=1}^n \left[\left(\frac{t_i' - \delta_i}{\theta_i - \delta_i} \right)^{\beta_i} - \left(\frac{t_i' + t_i - \delta_i}{\theta_i - \delta_i} \right)^{\beta_i} \right] \right\}$$

Now let $\delta_i=0$ and $\beta_i=1$ (exponential failures) for all i . Then

$$P_s(t_i) = \exp \left\{ \sum_{i=1}^n \left[\left(\frac{t_i'}{\theta_i} \right) - \left(\frac{t_i' + t_i}{\theta_i} \right) \right] \right\}$$

$$= \exp \left(-t_i \sum_{i=1}^n \frac{1}{\theta_i} \right)$$

Thus for a given set of scale parameters, θ_i , there exists a unique relationship between $P_s(t_i)$ and t_i .

For an individual component, k , the probability that the component survives past $t+t_i$ is

$$P_k(t_i) = \exp (-t_i/\theta_k)$$

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